Ambidexterity

November 17, 2024

The main references for the topics covered in the reading seminar are the following [HL13; CSY22].

The Talbot 2020 website has notes for some of the talks; in particular, see the syllabus for many relevant references (`https://math.mit.edu/events/talbot/index.php?year=2020&sub=talks`).

Talks

1. **Intro talk.** Overview of chromatic homotopy theory, outline of seminar.
2. **Categorical notions.** In the first half, give the construction of norms and integration [HL13, §4], [CSY22, §2-3], and basic properties that we'll use. Define the notion of $\mathcal{C}$-ambidexterity and m-semiadditivity. Then go through the base computation of $K(n)_*BC_p$ [RW80, §5]. Alternatively, you could use [CM17] (which is short but requires the Bousfield-Kuhn functor and Kahn-Priddy theorem). If you have time, you could discuss semiadditivity as a property [Har20].
3. **Bootstrap machine.** Following the plan in [CSY22, §1.3], explain how we'll reduce the proof of $T(n)$-local ambidexterity to the computation of $K(n)_*B^mC_p$. Go through the proofs of these steps [CSY22, §3-4], focusing on the construction of the operations α and δ.
4. **Ravenel-Wilson I.** The goal for the next two talks is to compute $K(n)_*B^mC_p$, by inducting on m. For this, step up the Dieudonné theory in [HL13, §1], and the classification of p-divisible groups according to their Dieudonné module. Then set up the Eilenberg-Steenrod spectral sequence, inductively computing $K(n)_*B^mC_p$ from knowledge of $K(n)_*B^{m-1}C_p$. We'll discuss this spectral sequence in detail in the next lecture.
5. **Ravenel-Wilson II.** Compute $K(n)_*B^mC_p$, as in [HL13, §2.4]. Conclude $K(n)$-local and $T(n)$-local ambidexterity, following [CSY22, §5].
6. **Applications (after Christmas): TBD.**

References

[CM17] Dustin Clausen and Akhil Mathew. "A Short Proof of Telescopic Tate Vanishing". In: *Proceedings of the American Mathematical Society* (2017). JSTOR: 90015415.

[CSY22] Shachar Carmeli, Tomer M. Schlank, and Lior Yanovski. "Ambidexterity in Chromatic Homotopy Theory". In: *Inventiones mathematicae* (June 2022).

[Har20] Yonatan Harpaz. "Ambidexterity and the Universality of Finite Spans". In: *Proceedings of the London Mathematical Society* (2020).

[HL13] Michael Hopkins and Jacob Lurie. *Ambidexterity in $K(n)$-Local Stable Homotopy Theory*. 2013.

[RW80] Douglas C. Ravenel and W. Stephen Wilson. "The Morava K-Theories of Eilenberg-MacLane Spaces and the Conner-Floyd Conjecture". In: *American Journal of Mathematics* (1980). JSTOR: 2374093.